RELATION OF EXPECT GAIN TO KELLY FORMULA

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The expected gain of a trading system – also known as *expected return* and referred to as *expectancy* by some authors – is related to the well-known Kelly formula in a straightforward way, as it is shown in this paper. Specifically, the Kelly formula for optimal bet size is the expected gain divided by the average winning trade, a ratio that maximizes geometric growth of profitable trading systems and betting strategies.

EXPECTED GAIN

A trading system is profitable over a period of time T, if the amount of winning trades is greater than the amount of losing trades over T. If we denote the amount of winning trades by the sum of winning trades and the amount of losing trades by the sum of losing trades, the following holds in the case of a profitable system:

$$\sum_T W - \sum_T L > 0 \tag{4.1}$$

The average winning trade is defined as the sum of the winning trades divided by their number:

$$\overline{W} = \frac{\sum_T W}{N_W} \tag{4.2}$$

where N_W is the number of winning trades. Similarly, the average losing trade is defined as:

$$\overline{L} = \frac{\sum_T L}{N_L} \tag{4.3}$$

where N_L is the number of losing trades. By combining equations 4.1, 4.2 and 4.3 we obtain

$$\overline{W} N_W - \overline{L} N_L > 0 \tag{4.4}$$

The number of winning trades plus the number of losing trades equals the total number of trades N, by definition, and thus:

$$N_L = N - N_W \tag{4.5}$$

After combining equations 4.4 and 4.5 and after dividing through by N > 0 we obtain:

$$\overline{W}\frac{N_W}{N} - \overline{L}\frac{N - N_W}{N} > 0 \tag{4.6}$$

Next, we define the profitability P -- also known as the *success rate* or *win rate* -- as the ratio of the number of wining trades N_W to the total number of trades N. Consequently, P is a fraction that ranges from 0 to 1 (or from 0 to 100% when expressed as a percentage):

$$P = \frac{N_W}{N} \tag{4.7}$$

After introducing equation 4.7 into equation 4.6, we obtain:

$$\overline{W}P - \overline{L}(1 - P) > 0 \tag{4.8}$$

The profitability P can be equated to the probability of win for sufficiently large samples of trades. In that case, equation 4.8 tells us that the probability of win times the average win minus the probability of loss times the average loss is always greater than zero for a profitable system. The difference of the weighted probabilities is defined as the expected gain E(g):

$$E(g) = \overline{W}P - \overline{L}(1 - P) \tag{4.9}$$

If we apply equation 4.9 in the case of tossing a fair coin, in which case P = 0.5 with an average win equal to the average loss, we get a zero value for the expected gain E(g). For a positive expected gain, it is the relation of profitability P to the ratio of average win to average loss R_{WL} that matters, not their specific values. As a result, high profitability does not suffice to have a winning trading system. For example, if P is equal to 0.8 (80% success rate), the average win is \$50 and the average loss is \$200, then from equation 4.9 it turns out that E(g) = \$50 x 0.8 – \$200 x (1 – 0.8) = 0, and therefore a trading method with these parameters is not going to be profitable over the medium and longer term.

THE KELLY FORMULA FOR OPTIMAL BET SIZE

The Kelly formula is based on the work of John kelly of Bell Laboratories in the 50's on the subject of telephone transmission signal to noise ratio. Professional gamblers and traders use a modified version of the original formula to determine optimal bet size, yet one can find several conflicting opinions in articles and online forums about its proper application, or even about the meaning and proper use of the result of the formula. The version of the original Kelly formula applicable to trading and betting systems is as follows:

$$\text{Kelly}\% = P - [(1 - P)/R_{WL}] \tag{4.10}$$

where Kelly % is the percentage of capital to risk per trade, P and R_{WL} are the profitability (success rate) and ratio of average win to average loss, respectively. The right hand side of equation 4.10 is the expected gain E(g) given by equation 4.9, divided by the average winning trade:

$$\text{Kelly}\,\% = \frac{\text{E(g)}}{\overline{\text{W}}} = \text{P} - [(1-\text{P})/\text{R}_{\text{WL}}] \qquad (4.11)$$

The expected gain E(g) represents the "edge", which is how much the trading system is expected to win on average, and the average winning trade is the "odds", which is the average amount won each time the trading system is profitable. Therefore, the optimal bet size according to the Kelly formula is equal to edge divided by odds. This is the ratio that is known to maximize geometric equity growth of a profitable trading system or betting strategy.

APPLICATION OF THE KELLY FORMULA

An application of the Kelly formula for optimal position size determination requires the actual values of the performance parameters of a trading system and specifically of the following two: the success rate (profitability) and the ratio of average win to average loss. This is a dramatic departure from other position sizing methods, such as the *fixed risk percent* method for example, because of the information that must available in advance before one can determine position size. Optimal position sizing based on the Kelly formula cannot be applied to newly developed trading systems because the actual values for the two parameters that are used by the formula are not available in advance. But even in the case of systems used in actual trading for an extended period of time, there is no guarantee that these parameters will remain constant or even within a certain range. This is especially true in the case of trend following systems, where the average win to average loss ratio is a random variable because it depends on the magnitude of unknown future trends and price volatility. Therefore, application of advanced optimal bet sizing methods, like the Kelly formula, requires frequent evaluation of system performance parameters in order to confirm that they maintain their minimum values.

Example: Stock Trading System

Trading capital: $100,000
Profitability: 65%
Ratio of average win to average loss: 1.25

In order to determine the optimal position size according to the Kelly formula, first we plug in the numbers in equation 4.10 to calculate Kelly %, the optimal bet size:

$$\text{Kelly}\,\% = 0.65 - (1 - 0.65)/1.25 = 0.37 \text{ or } 37\%$$

If the stop loss is 10% and the entry price is $85, then the number of shares according to the optimal betting strategy of the Kelly formula is:

$$\text{Number of shares N} = (0.37 \times \$100{,}000)/(0.10 \times \$85) = 4{,}352 \text{ or } 4{,}300 \text{ shares}$$

If the stock drops 10% then the loss will be equal to: 4,300 x 85 x 0.10 = $36,550 close to the allowable bet size due to the rounding of the number of shares.

However, the optimal bet size calculated above results in a number of shares that cannot be purchased with the available capital of 100K. This is because 4,300 shares x $85 per share = $365,500. In this case then, only $100,000/$85 = 1,176 or 1,100 shares can be purchased. The actual risk if the stock drops 10% is 1,100 x 85 x 0.1/100,000 = 0.0935, or 9.35%, instead of the 37% risk calculated by the Kelly formula.

Therefore, the optimal bet size can be realized provided that the ratio of Kelly % to stop-loss percent is less or equal to 1. In the above example this ratio is 37%/10% = 3.7. This means that only 27.02% of the optimal bet size can be used, or in another sense, one could say that the allowable solution is sub-optimal by a factor of about 3.7.

In comparison, a *fixed fractional* or *fixed risk percent* position sizing method for a system with the parameters of the example presented and a risk percent per trade equal to 4% limits the maximum number of shares to 465 only (just replace 0.37 above by 0.04 to get this result).

SUMMARY

The Kelly formula can often result in excessive risk per trade, in exchange for a geometric growth of equity performance. The excessive risk is justified based on the assumption that the profitability and ratio of average win to average loss estimates used in the formula correspond to the figures of the system. It is up to the trader to decide which method to use given his personal risk profile and confidence in his trading methods and systems. For beginners the fixed fractional position sizing method with a maximum risk per trade of no more than 1% is recommended until the performance of the method or system used is measured under actual market conditions. All traders, especially new ones, should always keep in mind that trading involves substantial financial risks and can result in total loss of capital.

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