

The Profitability Rule

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The Profitability Rule (see ref. 1) is an equation that relates the profitability (also known as *success rate* or *win rate*) of a trading system to the ratio of average winning to average losing trade and profit factor. The profitability of a trading system is defined here as the ratio of winning trades to total trades and ranges between 0 and 1 or, as a percent value, between 0 and 100. The profit factor is equal to the ratio of the amount of winning trades to the amount of losing trades. The average win is equal to the amount of winning trades divided by the number of winning number and, similarly, the average loss is equal to the amount of losing trades divided by the number of losing trades.

Derivation of the Profitability Rule

The average winning trade is equal to the sum of winning trades divided by the number of winning trades as follows:

$$\bar{W} = \frac{\sum W}{N_w} \quad (1)$$

where N_w is the number of winning trades. Similarly, the average losing trade is given by:

$$\bar{L} = \frac{\sum L}{N_L} \quad (2)$$

where N_L is the number of losing trades. The profit factor P_f is defined as the ratio of the amount of winning trades to the amount of losing trades

$$P_f = \frac{\sum W}{\sum L} \quad (3)$$

After combining equations (1), (2) and (3) we obtain

$$P_f = \frac{N_w \bar{W}}{N_L \bar{L}} \quad (4)$$

The sum of the number of winning trades plus the number of losing trades, is the total number of trades N by definition and thus N_L is equal to

$$N_L = N - N_w \quad (5)$$

Next, from the definition of the ratio of average winning to average losing trade, R_{WL} , we have:

$$R_{WL} = \frac{\overline{W}}{\overline{L}} \quad (6)$$

After introducing equations (5) and (6) into equation (4) we obtain

$$P_f = \frac{N_w}{N - N_w} R_{WL} \quad (7)$$

Next, we divide the numerator and denominator of equation (7) by N

$$P_f = \frac{\frac{N_w}{N}}{1 - \frac{N_w}{N}} R_{WL} \quad (8)$$

The profitability P of a trading system is defined as the ratio of the number of winning trades to the total number of trades. P takes values in the range 0 and 1 (or 0% to 100%):

$$P = \frac{N_w}{N} \quad (9)$$

After combining equations (8) and (9), we get the following expression for P_f

$$P_f = \frac{P}{1 - P} R_{WL} \quad (10)$$

Finally, after solving for P in equation (10) above we obtain the result

$$\boxed{P = \frac{1}{1 + \frac{R_{WL}}{P_f}} \quad \text{or} \quad P = \frac{P_f}{P_f + R_{WL}}} \quad (11)$$

Equation (11) is the Profitability Rule. It is an equation that relates the profitability P to the profit factor P_f and ratio of average winning to average losing trade R_{WL} . Given the values of the profit factor and ratio of average winning to average losing trade, using equation (11) one can determine the corresponding profitability. If the values of the profitability and ratio of average winning to

average losing trade are known instead, then equation (10) can be used to calculate the resulting profit factor. Alternatively, equation (10) can be solved for the ratio of average winning to average losing trade required given the values of profitability and profit factor, as follows

$$R_{WL} = \frac{1-P}{P} P_f \tag{12}$$

Figure 1 shows a 3-D graph of equation (11). The profitability P is plotted versus R_{WL} and P_f , for values ranging from 0 to 10.

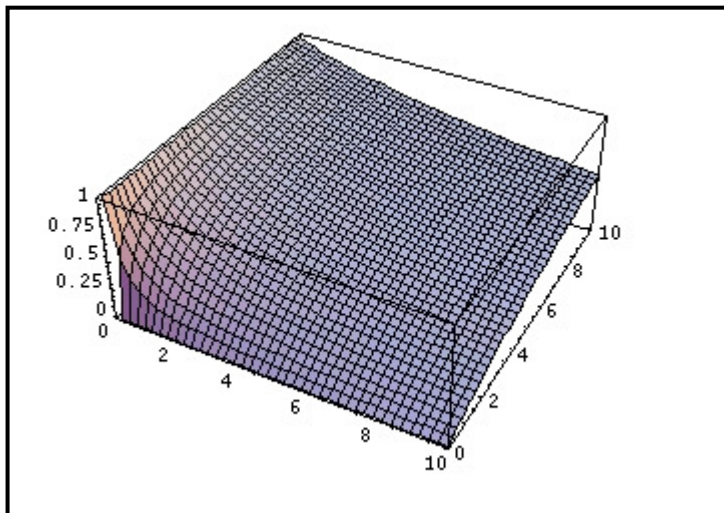


Figure 1: A 3-D graph of the Profitability Rule. Source: Mathematica.

Figure 2 shows a 2-D graph of the profitability P versus a single parameter, which is equal to the ratio R_{WL} / P_f . It may be seen from the graph in Figure 2 that the required profitability increases fast as the value of the ratio R_{WL} / P_f drops below 4 but it is comparatively low for values of the ratio much larger than 4.

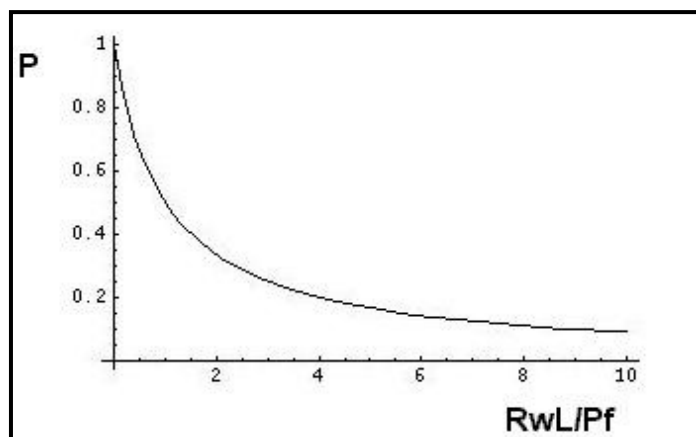


Figure 2: A 2-D graph of the Profitability Rule. Source: Mathematica

R_{WL}	Required Profitability P (x100)
10	16.67%
5	28.57%
2	50.00%
1	66.67%
0.5	80.00%
0.25	88.88%
0.125	94.11%

Table 1: Required profitability as a function of the ratio of average winning to average losing trade for a profit factor equal to 2.

Table 1 shows the required profitability of a trading system for various values of the R_{WL} parameter, computed using equation (11). The profit factor is set equal to 2. As R_{WL} increases the required profitability of a trading system decreases for a constant profit factor. Regarding the parameter R_{WL} , the following observations can be made from equation (11):

(1): Trading systems with low R_{WL} values must have high profitability, which implies a much larger number of winning trades than losing trades. This is the case with intraday or scalping trading systems. Intraday traders must have a high rate of success to stay profitable over the longer term.

(2): Trading systems with high R_{WL} values can have low profitability, meaning that they can generate fewer winning trades than losing trades and still stay profitable. This is the case with trend following systems that have a few large winners and many small losers.

(3): The ratio R_{WL}/P_f in equation (11) is an important parameter. It is evident that values of the profit factor P_f much greater than 1 reduce the effectiveness of higher R_{WL} values and thus increase the required profitability.

R_{WL}	Profit Factor P_f	Required Profitability P%
10	2	16.67
10	3	23.07
5	2	28.57
5	3	37.50
2	2	50.00
2	3	60.00
1	2	66.67
1	3	75.00
0.5	2	80.00
0.5	3	85.71

Table 2: Required profitability for various values of the ratio of average winning to average losing trade and profit factor

Table 2 shows values of the required profitability for various values of the ratio of average winning to average losing trade and profit factor. For each value of the ratio R_{WL} , two different values of the profit factor P_f are considered to illustrate the effect of the increased demand for higher profitability. This effect is also illustrated in Table 3 where the influence of the profit factor on profitability is considered for a constant R_{WL} value of 1. Notice, for example, that for a profit factor equal to 2 and a ratio of average winning to average losing trade equal to 1, the required profitability is 66.67%. If the profit factor is increased to 3, the required profitability is increased to 75.00%.

P_f	Minimum Required Profitability P (x100)
1.00	50.00%
1.25	55.56%
1.50	60.00%
1.75	63.64%
2.00	66.67%
2.50	71.43%
3.00	75.00%

Table 3: Required minimum profitability as a function of the profit factor for a constant ratio of average winning to average losing trade equal to 1.

The Profitability Rule and Trading Time Frames

Typical value of the parameter R_{WL} , the ratio of average winning to average losing trade, in the case of short-term or intraday trading systems are between 0.25 and 2. If the profitability (success rate) is maintained above its required value, as calculated by the Profitability Rule of equation (11), the system will always perform according to expectations. Trend following systems on the other hand require R_{WL} values much greater than 3. This is due to the fact that in trend following systems the objective is to minimize losses that occur during sideways markets and maximize profits realized during price trends.

However, in the case of trend following systems, future R_{WL} values are inherently uncertain and may not match any values obtained during back testing. This is because the magnitude of future price trends is unpredictable and at the same time the performance of position exit methods used depends on price volatility. Since both the magnitude of future trends and associated volatility cannot be known in advance, there is a possibility that future values of the parameter R_{WL} may be lower than the ones obtained from back testing. Therefore, in the context of the Profitability Rule, a given level of profitability of a trend following system could guarantee a minimum profit factor only if future trends have comparable magnitude as in the past and the same or lower price volatility to guarantee the effectiveness of position exit methods. This conclusion arises from the stochastic nature of the Profitability Rule in the case of trend following systems, in which case the parameter R_{WL} is a random variable. This also explains why profitable trend-following systems based on back testing often fail when used in actual trading.

Next, we will look at three examples of the use of the Profitability Rule.

Example 1: A short-term position trader of E-mini futures realizes a net profit of \$1000 in each winning trade (\$1000 target) and a net loss of \$500 in each losing trade (\$500 stop-loss). The trader is profitable 7 times out of ten over a sufficiently long period of time. (a) What is the trader's profit

factor? (b) By what factor the actual profitability exceeds the required profitability for break-even performance?

(a) First we calculate the ratio of average winning to average losing trade as follows:

$$R_{WL} = \$1000/\$500 = 2.0$$

Since $P = 70\%$, from equation (10) can we calculate the profit factor P_f

$$P_f = (0.7 \times 2.0) / (1.0 - 0.7) = 4.67$$

Thus, the amount of winning trades will be 4.67 times the amount of losing trades over a sufficiently long period of time.

(b) The required profitability for break-even performance (amount of winners = amount of losers) is found by applying equation (11) with $P_f = 1$.

$$P_{\min} = 100 / (1 + 2.0) = 33.33 \%$$

The trader's profitability P exceeds P_{\min} by a factor equal to

$$100 \times (70\% - 33.33\%) / 33.33\% = 110\%.$$

Example 2: System A is as profitable as system B but A has three times the ratio of average winning to average losing trade of B. How do their profit factors compare?

For system A:

$$P_{fA} = [P / (P - 1)] \times R_{WLA}$$

Similarly for trader B;

$$P_{fB} = [P / (P - 1)] \times R_{WLB}$$

After dividing P_{fA} by P_{fB} we get

$$P_{fA} / P_{fB} = R_{WLA} / R_{WLB}$$

Since $R_{WLB} = 3 \times R_{WLA}$ we obtain:

$$P_{fA} / P_{fB} = 1/3$$

Thus, **if the profitability stays constant, any increase in the ratio of average winning to average losing trade results in a proportional increase in the profit factor.**

Example 3: A trend following system has profitability P equal to 30% and uses a stop-loss of \$1,000. How large should the average winning trade be so that he will end up with a profit factor equal to 4?

From equation (12) we obtain:

$$R_{WL} = [(1-P)/P] \times Pf = [(1-0.3)/0.3] \times 4.0 = 9.33$$

$$R_{WL} = \text{avg. win}/\text{avg. loss} = \text{avg. win} / \$1,000 = 9.33$$

Result: avg. win = $9.33 \times \$1,000 = \$9,333$.

Thus, the average winning trade must be at least \$9,333 to maintain a profit factor of 4.0 when the profitability stays at 30% and the average loss is \$1,000.

Note that if the profitability were to increase to 40%, then:

$$R_{WL} = [(1-P)/P] \times Pf = [(1-0.4)/0.4] \times 4.0 = 6.0$$

$$R_{WL} = \text{avg. win}/\text{avg. loss} = \text{avg. win} / \$1,000 = 6.0$$

Result: avg. win = \$6,000

Thus an increase in the profitability by a factor of 1.33 reduces the required avg. win by a factor of 1.56 (\$9,333/\$6,000).

Comments

One should always keep in mind that under actual market conditions commissions and slippage can adversely affect the expected performance of trading systems. Essentially, what is affected is the average win to average loss ratio. The actual profitability may end up dropping below the expected level or even below the breakeven value in some cases. In order to account for the influence of unknown or random factors it is recommended to include a ‘safety factor’ in equation (11) as follows:

$$P = \frac{P_f}{P_f + \alpha R_{WL}}$$

where the safety factor α takes values between 0 and 1. In the case of intraday and short-term trading systems, values for α in the range 0.7 to 0.9 are recommended. In the case of trend following systems, since future R_{WL} values are random, lower values for the safety factor are recommended in the range 0.3 to 0.7.

The Profitability Rule is an equation that relates the profitability of a trading system to the ratio of average winning to average losing trade and profit factor. The equation derived in this article plays an important role in the analysis and synthesis of trading systems and exposes their performance limitations. The profitability rule provides, amongst other things, a quantitative means of evaluating alternative systems with different risk/reward and performance parameters. After all, it is not the absolute values of the risk/reward ratio, profitability and profit factor that are important, but the way they are related, as the Profitability Rule tells us in the form of an elegant equation. Regardless of the methodology used to develop trading systems, all traders should remember that trading involves high

financial risks and it can result in total loss of capital and it is suitable only to those that can afford to lose.

References

Profitability and Systematic Trading (Wiley, 2009). For more details see:

<http://www.wiley.com/WileyCDA/WileyTitle/productCd-047022908X.html>

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